Math Virtual Learning

Algebra IIB

Contracting a Logarithmic Expression

April 15, 2020

Algebra IIB
Lesson: April 14, 2020

Objective/Learning Target:
Students will contract a logarithmic function based on logarithmic rules.

Let's Get Started:

BELLWORK: lets review expanding logs from yesterday

1. Expand: $\log_3(5x)$

2. Expand: $\log_4\left(\dfrac{xy}{z}\right)$

3: Expand: $\ln(4a^5)$

4. Expand: $6\log_2\left(\dfrac{m^5}{x^2y}\right)$

Bellwork Answers:

1.   Because the log has 2 terms that are multiplied, separate them
     into addition: $\log_3 5 + \log_3 x$

2. Because the log has a division, separate each term, adding the
    terms in the numerator and subtracting the term in the
    denominator: $\log_4 x + \log_4 y - \log_4 z$

3. On the term that has an exponent, move the exponent to the front
    of the log: $\ln(4) + 5\ln(a)$

4. Using all the rules:
    $6(\log_2 m^5 - \log_2 x^2 - \log_2 y)$
    $6(5\log_2 m - 2\log_2 x - \log_2 y)$
    $30\log_2 m - 12\log_2 x - 6\log_2 y$

Use the same log rules, going from the expanded form to the condensed form

Logarithm Rules
(condensed)   (expanded)

1. $\log_a xy = \log_a x + \log_a y$

2. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$

3. $\log_a x^n = n\log_a x$

$\log_a b = \dfrac{\log_c b}{\log_c a}$

Examples

Condense: $\log_5 y + \log_5 z$ $\log_5 y \cdot z$

Condense: $\log_4 c - \log_4 d$ $\log_4 \left(\dfrac{c}{d}\right)$

Condense: $2\log_{11} h$ $\log_{11} h^2$

Watch this video for more examples of how to condense a logarithm:
Condensing Logarithmic Expressions

Let's practice using multiple rules:

Problem 1

$$\log_3 x + 2\log_3 y - 3\log_3 z$$

Step 1: Change coefficients to exponents.

Step 2: Combine the 1st 2 logs by multiplying and combine the final log by dividing.

Problem 2

$$3\log_8 x - (\log_8(4) + \log_8 x)$$

Step 1: Change coefficients to exponents.

Step 2: Combine the logs inside the parentheses by multiplying

Step 3: Combine the remaining logs by dividing.

Problem 3

$$\log_3(4) - 2\log_3 x + \log_3(5)$$

Step 1: Change coefficients to exponents.

Step 2: Combine the 1st 2 logs by dividing

Step 3: Combine remaining logs by multiplying.

Question: Does order matter? Look at question 1 and 3. Did the order of adding and subtracting the terms matter?

Practice Answers

Problem 1

$$\log_3 x + 2\log_3 y - 3\log_3 z$$

Step 1: Change coefficients to exponents.

$$\log_3 x + \log_3 y^2 - \log_3 z^3$$

Step 2: Combine the 1st 2 logs by multiplying and combine the final log by dividing.

$$\log_3 xy^2/z^3$$

Problem 2

$$3\log_8 x - (\log_8(4) + \log_8 x)$$

Step 1: Change coefficients to exponents.

$$\log_8 x^3 - (\log_8 4 + \log_8 x)$$

Step 2: Combine the logs inside the parentheses by multiplying

$$\log_8 x^3 - (\log_8 4x)$$

Step 3: Combine the remaining logs by dividing.

$$\log_8 x^3/4x$$

Problem 3

$$\log_3(4) - 2\log_3 x + \log_3(5)$$

Step 1: Change coefficients to exponents.

$$\log_3 4 - \log_3 x^2 + \log_3 5$$

Step 2: Combine the 1st 2 logs by dividing

$$\log_3 4/x^2 + \log_3 5$$

Step 3: Combine remaining logs by multiplying.

$$\log_3 20/x^2$$

Question: Does order matter? Look at question 1 and 3. Did the order of adding and subtracting the terms matter? Multiply and dividing are commutative so order does not matter

Condense each expression to a single logarithm.

13) $\log 3 - \log 8$

14) $\dfrac{\log 6}{3}$

15) $4\log 3 - 4\log 8$

16) $\log 2 + \log 11 + \log 7$

17) $\log 7 - 2\log 12$

18) $\dfrac{2\log 7}{3}$

19) $6\log_3 u + 6\log_3 v$

20) $\ln x - 4\ln y$

21) $\log_4 u - 6\log_4 v$

22) $\log_3 u - 5\log_3 v$

23) $20\log_6 u + 5\log_6 v$

24) $4\log_3 u - 20\log_3 v$

ANSWERS

13) $\log 3 - \log 8$

$\log \dfrac{3}{8}$

14) $\dfrac{\log 6}{3}$

$\log \sqrt[3]{6}$

19) $6\log_3 u + 6\log_3 v$

$\log_3 \left(v^6 u^6\right)$

20) $\ln x - 4\ln y$

$\ln \dfrac{x}{y^4}$

15) $4\log 3 - 4\log 8$

$\log \dfrac{3^4}{8^4}$

16) $\log 2 + \log 11 + \log 7$

$\log 154$

21) $\log_4 u - 6\log_4 v$

$\log_4 \dfrac{u}{v^6}$

22) $\log_3 u - 5\log_3 v$

$\log_3 \dfrac{u}{v^5}$

17) $\log 7 - 2\log 12$

$\log \dfrac{7}{12^2}$

18) $\dfrac{2\log 7}{3}$

$\log \sqrt[3]{7^2}$

23) $20\log_6 u + 5\log_6 v$

$\log_6 \left(v^5 u^{20}\right)$

24) $4\log_3 u - 20\log_3 v$

$\log_3 \dfrac{u^4}{v^{20}}$

Challenge Problems

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

26) $\log x \cdot \log 2$

Challenge Answers

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

$$\log \frac{4x^2}{75y^2}$$

26) $\log x \cdot \log 2$

Can't be simplified.